

Nuclear matter response function

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 (Received 7 November 1988)

The response function of nuclear matter is determined from experimental data on inclusive electron scattering from finite nuclei.

I. INTRODUCTION

The response function of a nucleus as measured by inclusive electron scattering provides important information on the nuclear ground state. This is true in particular if the cross sections $\sigma(q, \omega)$ cover a large range of momentum transfer q and energy loss ω . At values of $\omega < q^2/2m$ the response function yields a measure of the nucleon momentum distribution; this distribution is of special interest at large values of the initial momenta k which are studied at large q and ω . At energy loss above the quasielastic peak region, the response function is sensitive to the role of nucleon resonances in the nuclear medium, and the contribution of multinucleon processes such as meson-exchange currents (MEC).

In the past, this response function has been studied in detail for the lightest nuclei, $A=2,3$, for which "exact" wave functions are available.¹ For heavier nuclei, $A > 3$, calculations for the ground-state wave functions are less reliable. Moreover, the treatment of the final state produced in the quasielastic scattering process is rather rudimentary; given the fact that the final-state interaction is stronger, for these heavier nuclei, comparison between experiment and calculation has not been very fruitful.

Nuclear matter is the other "nucleus" for which calculations of quality comparable to the ones for $A=2,3$ are feasible. Sophisticated techniques have been developed for the treatment of nuclear matter, and the final state produced in quasielastic scattering can be described with an accuracy comparable to the one for the initial state. Several recent calculations have furnished predictions for the nuclear matter response function. We cite here the work of Fantoni and Pandharipande,² who perform a variational calculation based on the Urbana $V_{14} + TNI$ interaction, and the calculation of Butler and Koonin³ based on the Brueckner-Goldstone theory and the Reid soft core interaction.

Unfortunately, experimental results for nuclear matter are not achievable. Any comparison between calculation and experiment suffers from the finite-nucleus effects present in data for a heavy nucleus. While the average density in the central region of nuclei approaches the "nuclear matter" value at comparatively low A , the nuclear surface has a non-negligible effect. Even for a heavy nucleus such as lead, more than half of the nucleons are in the surface region where the nuclear density is between the "nuclear matter" value and zero. Such a large fraction of surface nucleons leads to important differences between heavy nuclei and nuclear matter. As shown by Casas *et al.*⁴ these differences are much more pronounced for the momentum distributions—of most interest for inclusive scattering—than for densities.

In a recent experiment we have measured⁵ the response function for nuclei with $A=4, 12, 27, 56$, and 197 over a large region of q with ω . In this paper we use these results to extrapolate to $A=\infty$, to provide an experimental determination of the response function of nuclear matter.

The extrapolation to nuclear matter requires an understanding of the competing mechanisms in this system, in particular the contribution from the excitation of internal degrees of freedom of the nucleons. Before discussing the extrapolation to $A=\infty$, we will discuss the contribution of nucleon excitations; for some applications, it is desirable to remove these effects.

II. REMOVAL OF NUCLEON EXCITATIONS

The response function measured by our experiment exhibits, at low q and ω , a peak corresponding to quasielastic scattering from individual nucleons. At large q, ω the peak corresponding to excitation of the Δ resonance is apparent, and at very large q, ω the cross sections are dominated by deep-inelastic scattering from a (bound) nucleon. At the highest momentum transfers, these contributions from inelastic processes on the nucleon are already important

near the top of the quasielastic peak, at $\omega \simeq q^2/2m$.

For some studies it may be desirable to remove these contributions. This is true in particular if one wants to compare experiment to nuclear response functions calculated in terms of nucleonic degrees of freedom only. We therefore will present two different response functions: (a) an extrapolation of the data $\sigma(q, \omega)$ including the quasielastic and inelastic pieces, and (b) an extrapolation of the data after subtracting the effects of nucleon excitations.

In order to calculate these inelastic processes on the nucleon, we start from the experimental data on the proton and neutron structure functions. Their dependence on q and ω has been parametrized⁶ and allows us to calculate the elementary structure functions for the q, ω of interest for our data. To calculate the corresponding response function of the nucleus, we fold the nucleon response functions according to the procedure described by Bodek and Ritchie.⁷ For the folding, we use the nucleon momentum distribution derived from a y scaling analysis⁸ of the data. Modifications of the nucleon inelastic response function due to nuclear binding (the EMC effect) are accounted for;⁹ the correction made reproduces the x -dependent ratio of nucleon/nucleus cross sections measured in Ref. 10.

The calculated cross sections for the inelastic excitation of the bound nucleon describe well the nuclear data in the region where they should dominate. Typical deviations are of order 10%. We subtract this contribution from the experimental data, in order to obtain a response function with inelastic processes on the nucleon removed. A careful study of the various approximations, and alternative calculations of the inelastic response and folding procedure, leads us to assign a 20% error to the calculated inelastic contribution. We do not quote the results if the subtraction amounts to more than 30%. For the values of the response function, the error of the extracted inelastic contribution is included.

The subtraction procedure yields reliable values for the structure function for $\omega < q^2/2m$. Given the approximate symmetry of the quasielastic peak around this value of ω , the subtracted data cover much of the region of interest for quasielastic scattering from nuclei. As indicated, the subtracted data have incoherent inelastic excitations of individual nucleons, π production, Δ excitation, etc. removed. We point out, however, that multinucleon effects, such as meson-exchange currents, still are present.

III. EXTRAPOLATION PROCEDURE

In impulse approximation (IA), the inclusive cross section $\sigma(q, \omega)$ receives contributions from several processes: quasielastic electron-nucleon scattering, excitation of the Δ , higher nucleon resonances, and deep-inelastic electron-nucleon scattering. The A nucleons contribute incoherently (contributions due to MEC will be discussed later). In the IA, the respective cross sections per nucleon are given as integrals over the nucleon spectral function S and the elementary nucleon structure functions (form factors).

To extrapolate from finite nuclei to nuclear matter, we start from the following consideration. To a good approximation the nuclear response function is an incoherent sum over the contributions from individual nucleons. The volume piece, proportional to the nuclear mass number A , is the one we are interested in when discussing nuclear matter. Effects of the nuclear surface are proportional to $A^{2/3}$ given the $A^{1/3}$ dependence of the nuclear radius and the fact that the nuclear surface thickness is largely independent of A . The ratio of surface to volume contributions thus is proportional to $A^{-1/3}$. Extrapolating the response function per nucleon to $A^{-1/3} = 0$ as a linear function of $A^{-1/3}$ gives the nuclear matter response.

This $A^{-1/3}$ can be derived more formally using the local density approximation. This approximation has been shown to be valid for many nuclear observables; it has been derived specifically in Ref. 18 for nucleon momentum distributions. In the local density approximation, we may consider $S(k, E, \rho)$ to be a quantity that, in addition to the usual dependence on initial momentum k and separation energy E , also depends on the local nuclear density $\rho(r)$. We then can write the nuclear cross section

$$\sigma(q, \omega) = \int S(\rho(r)) F d\mathbf{k} dE \rho(r) dr. \quad (1)$$

The factor F , which contains the nucleon structure function and all the kinematical factors, depends on k, E, q, ω , and is of no interest for the following discussion.

To make explicit the dependence of $\sigma(q, \omega)$ on the nuclear mass number A we split the density into two terms $\rho_c + \rho_s$. The former corresponds to the idealized hard-sphere density

$$\rho_c(r < R_0) = \rho_0, \quad \rho_c(r > R_0) = 0,$$

the latter corresponds to a surface-peaked distribution (with total volume zero) that describes the difference between ρ_c and the real density $\rho(r)$. From elastic electron scattering we know that ρ_c is largely independent of A , with $R_0 = r_0 A^{1/3}$. We also know that the quantity $\rho(r) - \rho_0$ is a nearly universal function of $R_0 - r$, which has a shape largely independent of A , and which is significantly different from zero only in the surface region. These two contributions to the density give different contributions to the nuclear response.

1. The nucleons in the constant-density region of the nucleus give the contribution of interest. The corresponding contribution to Eq. (1), integrated over the constant-density region, gives

$$\sigma_c(q, \omega) = A \int S(\rho_0) F d\mathbf{k} dE. \quad (2)$$

The quantity σ_c/A in the limit $A \rightarrow \infty$ is the nuclear matter response per nucleon.

2. The nucleons in the surface region contribute differently due to the change in S between densities of ρ_0 and $\rho = 0$. Given that the radial dependence of $\rho(r)$ in the surface region is a near-universal function of $r - R_0$, and that for large A the region where $\rho(r) - \rho_0 \neq 0$ is small compared to R_0 , the angular part of the integral over ρ can be carried out

$$\sigma_s(q, \omega) = A^{2/3} 4\pi r_0^2 \int S(\rho(r)) F d\mathbf{k} dE \rho_s(r) dr. \quad (3)$$

This contribution represents the difference between nucleons with the idealized density distribution ρ_c and nucleons with a density having finite surface thickness.

The total nuclear response, divided by A , then reads

$$\begin{aligned} \sigma(q, \omega)/A &= \sigma_c(q, \omega)/A + \sigma_s(q, \omega)/A \\ &= \int S(\rho_0) F d\mathbf{k} dE \\ &\quad + A^{-1/3} \int S(\rho(r)) F d\mathbf{k} dE 4\pi r_0^2 \rho_s(r) dr, \end{aligned} \quad (4)$$

with all of the first-order A dependence explicitly shown.

Eq. (4) shows that, in agreement with the simple argument made earlier, the nuclear response $\sigma(q, \omega)/A$ is expected to be a linear function of $A^{-1/3}$. This finding agrees with the $A^{-1/3}$ dependence found in theoretical calculations of momentum distributions of finite systems.¹¹ Extrapolating the data $\sigma_{\text{exp}}(q, \omega)/A \equiv \Sigma(q, \omega)$ as a linear function of $A^{-1/3}$ to 0 ($A = \infty$) yields the nuclear matter response.

To fix these ideas, we show in Fig. 1 an example for an extrapolation as a function of $A^{-1/3}$. Disregarding the nucleus ${}^4\text{He}$ ($A^{-1/3} = 0.63$), the nuclear response $\Sigma(q, \omega)$ for $A = 12-197$ is well described by a linear function of $A^{-1/3}$. When extrapolated to zero, this gives the nuclear matter result. The corresponding plot for an extrapolation as a function of A is given in Fig. 2. This figure shows that extrapolation as a function of A is more difficult, although the curve seems to convey better the idea of "saturation." Due to the large fraction of surface nucleons (50% for the largest A), even heavy nuclei significantly differ from nuclear matter. In Sec. IV we show additional examples for $A^{-1/3}$ extrapolations for several extreme cases of q, ω . Comparison of Figs. 1 and 2 clearly shows that the response does not saturate at the

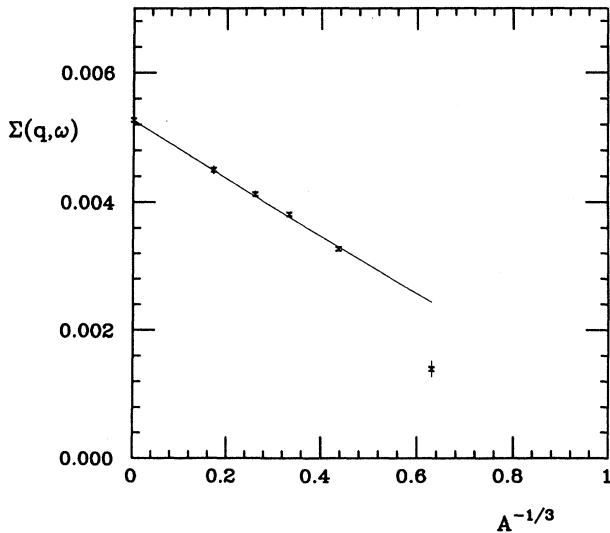


FIG. 1. Response function per nucleon as a function of $A^{-1/3}$, for $E = 3.6$ GeV, $\theta = 16^\circ$, and $\omega = 180$ MeV. Only the points $A^{-1/3}$ less than 0.5 ($A \geq 12$) are used for the extrapolation.

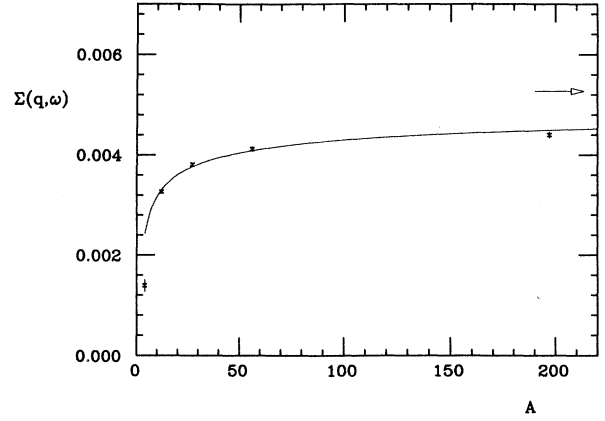


FIG. 2. Same as Fig. 1, but plotted as function of A . The nuclear matter value is indicated by an arrow.

largest A accessible.

The extrapolation is performed in slightly different ways for the two cases of interest:

(a) To obtain the total nuclear matter response function, with no correction for inelastic processes, we divide the nuclear cross sections $\sigma(q, \omega)$ by A , and then extrapolate as a function of $A^{-1/3}$.

$$\Sigma(q, \omega) = \sigma(q, \omega)/A. \quad (5)$$

(b) To obtain the nuclear matter response function with excitations of the nucleons removed, we make use of additional knowledge of the response function. For quasi-elastic scattering, the relative contribution of protons and neutrons changes as a function of A . Although the protons dominate due to the larger electron-proton cross section, the contribution of neutrons is not negligible. For the extrapolation, we assume that the spectral function for protons and neutrons is the same. We remove the trivial dependence on N, Z by extrapolating the quantity

$$\Sigma(q, \omega) = \sigma(q, \omega) / (Z\sigma_{ep} + N\sigma_{en}) \cdot (\sigma_{ep} + \sigma_{en})/2 \quad (6)$$

where σ_{eN} are the elastic electron-nucleon cross sections. We produce results independent of the specific choice of $\sigma_{eN}(q)$ by multiplying the extrapolated $\Sigma(q, \omega)$ with $(\sigma_{ep} + \sigma_{en})/2$. This gives values of the cross section per nucleon for symmetric nuclear matter.

The structure function just calculated may depend on the validity of the assumption that the response functions are identical for protons and neutrons. Although estimates show that this does not introduce a significant error, we consider an alternative. The value of $\Sigma(q, \omega)$ obtained via the extrapolation of Eq. (6) implicitly corresponds to nuclear matter which has a proton/neutron ratio extrapolated from the one of finite nuclei. In a more careful treatment, one therefore might want to drop the assumption of identical proton and neutron response functions. This can be done by multiplying the extrapolated value of Σ by $(\sigma_{ep} + R\sigma_{en})/2$. The resulting response function then would correspond to nuclear matter with $R = (N/Z)_\infty = 1.85$ (extrapolated from stable nuclei). In the following, we will list the numerical

results for the case $N=Z$. The results for $N \neq Z$ are available from the first author.

In the derivation of the $A^{-1/3}$ dependence of the nuclear response, we have not considered processes like MEC, which may give contributions of order 10%. These processes involve two nucleons, and could depend on A in a complicated way. MEC processes, however, are of short range; for the heavier nuclei of main interest here, their range is much shorter than R_0 . In this case MEC contributions to $\sigma_0(q, \omega)$ are treated correctly in Eq. (4); contributions from nucleons in the surface are extrapolated to zero like the one-body contributions.

In the extrapolation to $A = \infty$ we have considered possible contributions that have a dependence on A that differs from the volume and surface terms already employed. There could be a residual dependence of the cross section on $A^{1/3}$ indicating a dependence on the nuclear radius. When extrapolating as a function of the variable $A^{-1/3}$ this would lead to a quadratic term in addition to the linear one used previously. We have tried such a quadratic extrapolation, but have found no need to include the quadratic term when extrapolating $A = 12-197$. ${}^4\text{He}$ is a singular nucleus in more than one respect, and cannot be included successfully with or without the quadratic term. When including the quadratic term for $A = 12-197$, the response function does not change in a systematic way; neighboring bins in ω show strongly fluctuating quadratic terms; on average no need for a quadratic term becomes apparent. We will not pursue such extrapolations further.

To verify whether the $A^{-1/3}$ extrapolation indeed corresponds to the A dependence of the data, we have performed fits of the data using an A^{-x} dependence, with variable x . We find that the χ^2 of fits averaged over all data have a minimum at $x = 0.32 \pm 0.03$. This confirms

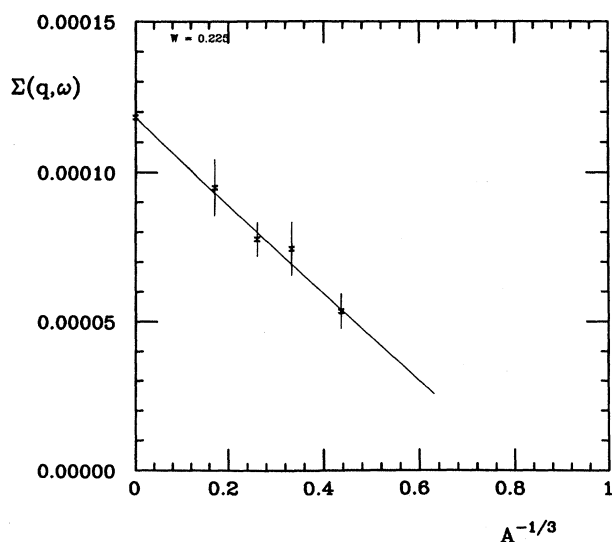


FIG. 3. Response function per nucleon as a function of $A^{-1/3}$, for $E = 3.6$ GeV, $\theta = 20^\circ$, and $\omega = 225$ MeV. Only the points $A^{-1/3}$ less than 0.5 ($A \geq 12$) are used for the extrapolation.

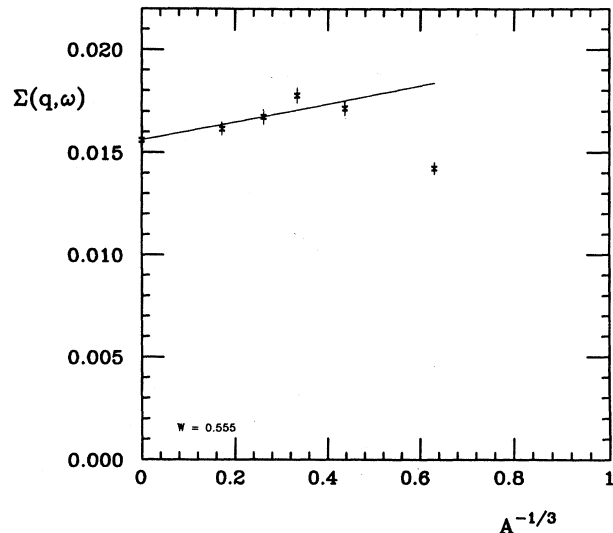


FIG. 4. Response function per nucleon as a function of $A^{-1/3}$, for $E = 3.6$ GeV, $\theta = 20^\circ$, and $\omega = 555$ MeV. Only the points $A^{-1/3}$ less than 0.5 ($A \geq 12$) are used for the extrapolation.

that the $A^{-1/3}$ in Eq. (4) is correct.

For momentum transfers $q^2 < 3.5$ (GeV/c)² our data cover basically all nuclei $A = 4-197$. For $q^2 = 3.5$ and the lowest values of ω , and for $q^2 > 3.5$ (GeV/c)², our data are essentially limited to the nucleus iron. The extrapolation in $A^{-1/3}$ previously described, therefore, cannot be applied to these highest- q^2 data. For studies of the nuclear matter momentum distribution at large momenta k it would be of great interest to provide data up

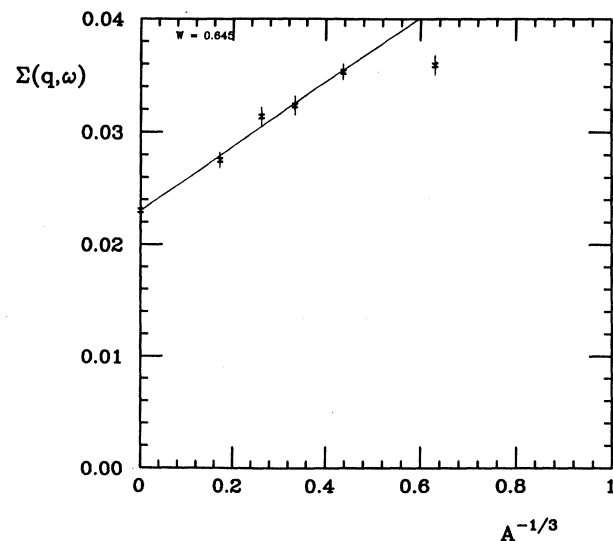


FIG. 5. Response function per nucleon as a function of $A^{-1/3}$, for $E = 3.6$ GeV, $\theta = 20^\circ$, and $\omega = 645$ MeV. Only the points $A^{-1/3}$ less than 0.5 ($A \geq 12$) are used for the extrapolation.

TABLE I. Nuclear matter response as a function of energy loss. The second column gives the cross sections defined in Eq. (5); the third column gives the cross sections, corrected for nucleon excitations, defined by Eq. (6). $E = 2.02$ GeV, $\theta = 15^\circ$.

ω (GeV)	$d^2\sigma/d\Omega d\omega$ ($\mu\text{b}/\text{sr GeV}$)	$d^2\sigma/d\Omega d\omega_{\text{sub}}$ ($\mu\text{b}/\text{sr, GeV}$)
0.060	$(0.163 \pm 0.009) \times 10$	$(0.166 \pm 0.009) \times 10$
0.075	$(0.218 \pm 0.007) \times 10$	$(0.232 \pm 0.007) \times 10$
0.090	$(0.257 \pm 0.008) \times 10$	$(0.278 \pm 0.008) \times 10$
0.105	$(0.293 \pm 0.018) \times 10$	$(0.324 \pm 0.019) \times 10$
0.120	$(0.334 \pm 0.019) \times 10$	$(0.369 \pm 0.020) \times 10$
0.135	$(0.305 \pm 0.015) \times 10$	$(0.341 \pm 0.015) \times 10$
0.150	$(0.321 \pm 0.015) \times 10$	$(0.358 \pm 0.015) \times 10$
0.165	$(0.333 \pm 0.022) \times 10$	$(0.371 \pm 0.023) \times 10$
0.180	$(0.345 \pm 0.021) \times 10$	$(0.383 \pm 0.022) \times 10$
0.195	$(0.338 \pm 0.019) \times 10$	$(0.374 \pm 0.020) \times 10$
0.210	$(0.337 \pm 0.013) \times 10$	$(0.370 \pm 0.013) \times 10$
0.225	$(0.323 \pm 0.012) \times 10$	$(0.353 \pm 0.013) \times 10$
0.240	$(0.321 \pm 0.014) \times 10$	$(0.346 \pm 0.015) \times 10$
0.255	$(0.316 \pm 0.014) \times 10$	$(0.337 \pm 0.014) \times 10$
0.270	$(0.325 \pm 0.013) \times 10$	$(0.341 \pm 0.014) \times 10$
0.285	$(0.290 \pm 0.010) \times 10$	$(0.298 \pm 0.011) \times 10$
0.300	$(0.294 \pm 0.016) \times 10$	$(0.293 \pm 0.018) \times 10$
0.315	$(0.264 \pm 0.015) \times 10$	$(0.251 \pm 0.019) \times 10$
0.330	$(0.242 \pm 0.015) \times 10$	$(0.219 \pm 0.022) \times 10$
0.345	$(0.257 \pm 0.015) \times 10$	$(0.220 \pm 0.026) \times 10$
0.360	$(0.267 \pm 0.016) \times 10$	$(0.211 \pm 0.032) \times 10$

TABLE II. Nuclear matter response as a function of energy loss. The second column gives the cross sections defined in Eq. (5); the third column gives the cross sections, corrected for nucleon excitations, defined by Eq. (6). $E = 2.02$ GeV, $\theta = 20^\circ$.

ω (GeV)	$d\sigma/d\Omega d\omega$ ($\mu\text{b}/\text{sr GeV}$)	$d\sigma/d\Omega d\omega_{\text{sub}}$ ($\mu\text{b}/\text{sr GeV}$)
0.060	$(0.353 \pm 0.034) \times 10^{-1}$	$(0.352 \pm 0.034) \times 10^{-1}$
0.075	$(0.718 \pm 0.030) \times 10^{-1}$	$(0.728 \pm 0.030) \times 10^{-1}$
0.090	(0.121 ± 0.004)	(0.125 ± 0.004)
0.105	(0.185 ± 0.005)	(0.198 ± 0.005)
0.120	(0.277 ± 0.021)	(0.296 ± 0.022)
0.135	(0.336 ± 0.024)	(0.361 ± 0.025)
0.150	(0.438 ± 0.021)	(0.470 ± 0.021)
0.165	(0.492 ± 0.023)	(0.532 ± 0.023)
0.180	(0.538 ± 0.022)	(0.582 ± 0.022)
0.195	(0.543 ± 0.023)	(0.592 ± 0.023)
0.210	(0.595 ± 0.031)	(0.694 ± 0.032)
0.225	(0.643 ± 0.027)	(0.701 ± 0.028)
0.240	(0.669 ± 0.034)	(0.727 ± 0.035)
0.255	(0.679 ± 0.028)	(0.737 ± 0.028)
0.270	(0.661 ± 0.033)	(0.716 ± 0.034)
0.285	(0.635 ± 0.032)	(0.683 ± 0.033)
0.300	(0.668 ± 0.032)	(0.707 ± 0.033)
0.315	(0.745 ± 0.042)	(0.774 ± 0.044)
0.330	(0.701 ± 0.028)	(0.711 ± 0.032)
0.345	(0.762 ± 0.037)	(0.749 ± 0.042)
0.360	(0.686 ± 0.035)	(0.644 ± 0.044)
0.375	(0.779 ± 0.034)	(0.710 ± 0.049)
0.390	(0.693 ± 0.023)	(0.584 ± 0.050)
0.405	(0.669 ± 0.023)	(0.554 ± 0.061)
0.420	(0.758 ± 0.031)	(0.576 ± 0.076)
0.435	(0.683 ± 0.031)	(0.464 ± 0.089)
0.450	(0.645 ± 0.033)	(0.393 ± 0.103)

TABLE III. Nuclear matter response as a function of energy loss. The second column gives the cross sections defined in Eq. (5); the third column gives the cross sections, corrected for nucleon excitations, defined by Eq. (6). $E = 3.595$ GeV, $\theta = 16^\circ$.

ω (GeV)	$d^2\sigma/d\Omega d\omega$ ($\mu\text{b/sr GeV}$)	$d^2\sigma/d\Omega d\omega_{\text{sub}}$ ($\mu\text{b/sr GeV}$)
0.090	$(0.508 \pm 0.035) \times 10^{-3}$	$(0.526 \pm 0.036) \times 10^{-3}$
0.105	$(0.972 \pm 0.029) \times 10^{-3}$	$(0.844 \pm 0.047) \times 10^{-3}$
0.120	$(0.142 \pm 0.004) \times 10^{-2}$	$(0.124 \pm 0.006) \times 10^{-2}$
0.135	$(0.216 \pm 0.005) \times 10^{-2}$	$(0.209 \pm 0.008) \times 10^{-2}$
0.150	$(0.296 \pm 0.006) \times 10^{-2}$	$(0.303 \pm 0.009) \times 10^{-2}$
0.165	$(0.403 \pm 0.007) \times 10^{-2}$	$(0.381 \pm 0.011) \times 10^{-2}$
0.180	$(0.526 \pm 0.008) \times 10^{-2}$	$(0.536 \pm 0.013) \times 10^{-2}$
0.195	$(0.716 \pm 0.010) \times 10^{-2}$	$(0.674 \pm 0.016) \times 10^{-2}$
0.210	$(0.108 \pm 0.006) \times 10^{-1}$	$(0.115 \pm 0.006) \times 10^{-1}$
0.225	$(0.134 \pm 0.007) \times 10^{-1}$	$(0.143 \pm 0.007) \times 10^{-1}$
0.240	$(0.165 \pm 0.008) \times 10^{-1}$	$(0.176 \pm 0.008) \times 10^{-1}$
0.255	$(0.211 \pm 0.009) \times 10^{-1}$	$(0.226 \pm 0.009) \times 10^{-1}$
0.270	$(0.290 \pm 0.011) \times 10^{-1}$	$(0.309 \pm 0.011) \times 10^{-1}$
0.285	$(0.385 \pm 0.012) \times 10^{-1}$	$(0.408 \pm 0.012) \times 10^{-1}$
0.300	$(0.482 \pm 0.013) \times 10^{-1}$	$(0.512 \pm 0.013) \times 10^{-1}$
0.315	$(0.576 \pm 0.014) \times 10^{-1}$	$(0.610 \pm 0.015) \times 10^{-1}$
0.330	$(0.727 \pm 0.045) \times 10^{-1}$	$(0.769 \pm 0.047) \times 10^{-1}$
0.345	$(0.863 \pm 0.050) \times 10^{-1}$	$(0.909 \pm 0.051) \times 10^{-1}$
0.360	$(0.985 \pm 0.054) \times 10^{-1}$	$(1.037 \pm 0.056) \times 10^{-1}$
0.375	(0.105 ± 0.006)	(0.111 ± 0.006)
0.390	(0.115 ± 0.006)	(0.119 ± 0.006)
0.405	(0.124 ± 0.005)	(0.128 ± 0.005)
0.420	(0.133 ± 0.005)	(0.135 ± 0.005)
0.435	(0.141 ± 0.005)	(0.142 ± 0.006)
0.450	(0.152 ± 0.007)	(0.151 ± 0.008)
0.465	(0.156 ± 0.007)	(0.152 ± 0.008)
0.480	(0.157 ± 0.007)	(0.147 ± 0.009)
0.495	(0.169 ± 0.007)	(0.155 ± 0.009)
0.510	(0.165 ± 0.007)	(0.143 ± 0.010)
0.525	(0.175 ± 0.006)	(0.151 ± 0.010)
0.540	(0.156 ± 0.006)	(0.124 ± 0.012)
0.555	(0.170 ± 0.006)	(0.132 ± 0.013)
0.570	(0.173 ± 0.011)	(0.124 ± 0.018)
0.585	(0.197 ± 0.011)	(0.137 ± 0.020)
0.600	(0.201 ± 0.011)	
0.615	(0.194 ± 0.011)	
0.630	(0.193 ± 0.011)	
0.645	(0.208 ± 0.008)	
0.660	(0.214 ± 0.009)	
0.675	(0.214 ± 0.008)	
0.690	(0.228 ± 0.012)	
0.705	(0.213 ± 0.012)	
0.720	(0.209 ± 0.013)	
0.735	(0.202 ± 0.012)	
0.750	(0.185 ± 0.012)	
0.765	(0.209 ± 0.009)	
0.780	(0.215 ± 0.009)	
0.795	(0.218 ± 0.009)	
0.810	(0.247 ± 0.012)	
0.825	(0.217 ± 0.013)	
0.840	(0.207 ± 0.012)	
0.855	(0.211 ± 0.012)	
0.870	(0.214 ± 0.009)	
0.885	(0.223 ± 0.010)	
0.900	(0.222 ± 0.010)	
0.915	(0.221 ± 0.015)	
0.930	(0.227 ± 0.014)	

TABLE III. (Continued).

ω (GeV)	$d^2\sigma/d\Omega d\omega$ ($\mu\text{b/sr GeV}$)	$d^2\sigma/d\Omega d\omega_{\text{sub}}$ ($\mu\text{b/sr GeV}$)
0.945	(0.231 \pm 0.015)	
0.960	(0.222 \pm 0.014)	
0.975	(0.239 \pm 0.010)	
0.990	(0.250 \pm 0.010)	
1.005	(0.207 \pm 0.013)	
1.020	(0.226 \pm 0.014)	
1.035	(0.232 \pm 0.014)	
1.050	(0.240 \pm 0.013)	
1.065	(0.232 \pm 0.014)	
1.080	(0.220 \pm 0.010)	
1.095	(0.240 \pm 0.010)	
1.110	(0.244 \pm 0.015)	
1.125	(0.220 \pm 0.014)	
1.140	(0.229 \pm 0.015)	
1.155	(0.209 \pm 0.015)	
1.170	(0.240 \pm 0.011)	
1.185	(0.221 \pm 0.010)	
1.200	(0.254 \pm 0.012)	
1.215	(0.219 \pm 0.012)	
1.230	(0.234 \pm 0.012)	
1.245	(0.260 \pm 0.012)	
1.260	(0.247 \pm 0.012)	
1.275	(0.226 \pm 0.009)	
1.290	(0.233 \pm 0.009)	
1.305	(0.234 \pm 0.014)	
1.320	(0.236 \pm 0.014)	
1.335	(0.230 \pm 0.014)	
1.350	(0.192 \pm 0.014)	
1.365	(0.229 \pm 0.010)	
1.380	(0.225 \pm 0.010)	
1.395	(0.229 \pm 0.015)	
1.410	(0.219 \pm 0.015)	
1.425	(0.214 \pm 0.014)	
1.440	(0.197 \pm 0.010)	
1.455	(0.225 \pm 0.010)	
1.470	(0.236 \pm 0.014)	
1.485	(0.228 \pm 0.014)	
1.500	(0.238 \pm 0.014)	
1.515	(0.213 \pm 0.014)	
1.530	(0.206 \pm 0.014)	
1.545	(0.218 \pm 0.014)	

to the largest q^2 possible. In the following, we describe a procedure that allows us to determine the nuclear matter response function for these highest q^2 as well.

The relative slope s

$$s = d\Sigma(q, \omega)/d(A^{-1/3})/\Sigma(q, \omega) \quad (7)$$

of the straight-line fits in $A^{-1/3}$ is a slowly varying function of energy loss. A plot of s as a function of the scaling variable⁸ y reveals that s depends on y only. The slope $s(y)$ depends very little on q^2 , with the exception of the lowest- ω points at the two lowest values of q^2 , where final-state interaction effects dominate.¹² Since the relative slope $s(y)$ at the higher values of q^2 is, to first order,

independent of q^2 , we can use this slope to extrapolate from iron to nuclear matter at the highest q^2 . When doing so, we attribute realistic errors to $s(y)$ which reflect the fluctuations and the residual change with q . These errors are incorporated in the calculation of the nuclear matter response function.

The effect of the Coulomb distortion on our inclusive cross sections is small due to the high incident electron energies of 2 and 3.6 GeV of this experiment. We nevertheless correct for the Coulomb distortion to avoid possible problems with the (implicit) extrapolation to $Z = \infty$. The corrections are performed using the effective-energy description employed for elastic scattering. The energies of ingoing and outgoing electrons are increased by the

Coulomb energy ΔE at the nuclear surface calculated using a uniform charge distribution, $\Delta E = 1.5Z\alpha/r_{eq}$. When using this correction the effective energies for different target nuclei are no longer the same. In order to perform the $A^{-1/3}$ extrapolation at constant E, ω we cal-

culate the change of $\Sigma(E, \omega)$ due to a change in E using the derivatives $\partial\Sigma/\partial E$ calculated from a model¹³ that fits our data over the full range of q, ω .

The change of the cross section due to Coulomb distortion amounts to 14% at maximum for gold. The

TABLE IV. Nuclear matter response as a function of energy loss. The second column gives the cross sections defined in Eq. (5); the third column gives the cross sections, corrected for nucleon excitations, defined by Eq. (6). $E = 3.595$ GeV, $\theta = 20^\circ$.

ω (GeV)	$d\sigma/d\Omega d\omega$ ($\mu\text{b/sr GeV}$)	$d\sigma/d\Omega d\omega_{\text{sub}}$ ($\mu\text{b/sr GeV}$)
0.165	$(0.205 \pm 0.052) \times 10^{-4}$	$(0.217 \pm 0.053) \times 10^{-4}$
0.180	$(0.259 \pm 0.063) \times 10^{-4}$	$(0.278 \pm 0.065) \times 10^{-4}$
0.195	$(0.595 \pm 0.085) \times 10^{-4}$	$(0.609 \pm 0.086) \times 10^{-4}$
0.210	$(0.738 \pm 0.096) \times 10^{-4}$	$(0.776 \pm 0.099) \times 10^{-4}$
0.225	$(0.118 \pm 0.012) \times 10^{-3}$	$(0.125 \pm 0.013) \times 10^{-3}$
0.240	$(0.174 \pm 0.016) \times 10^{-3}$	$(0.183 \pm 0.016) \times 10^{-3}$
0.255	$(0.246 \pm 0.019) \times 10^{-3}$	$(0.259 \pm 0.019) \times 10^{-3}$
0.270	$(0.301 \pm 0.018) \times 10^{-3}$	$(0.317 \pm 0.019) \times 10^{-3}$
0.285	$(0.404 \pm 0.021) \times 10^{-3}$	$(0.426 \pm 0.021) \times 10^{-3}$
0.300	$(0.485 \pm 0.023) \times 10^{-3}$	$(0.512 \pm 0.024) \times 10^{-3}$
0.315	$(0.739 \pm 0.050) \times 10^{-3}$	$(0.774 \pm 0.051) \times 10^{-3}$
0.330	$(0.873 \pm 0.055) \times 10^{-3}$	$(0.914 \pm 0.057) \times 10^{-3}$
0.345	$(0.117 \pm 0.006) \times 10^{-2}$	$(0.122 \pm 0.006) \times 10^{-2}$
0.360	$(0.140 \pm 0.007) \times 10^{-2}$	$(0.145 \pm 0.007) \times 10^{-2}$
0.375	$(0.164 \pm 0.008) \times 10^{-2}$	$(0.169 \pm 0.008) \times 10^{-2}$
0.390	$(0.211 \pm 0.009) \times 10^{-2}$	$(0.217 \pm 0.009) \times 10^{-2}$
0.405	$(0.259 \pm 0.010) \times 10^{-2}$	$(0.263 \pm 0.010) \times 10^{-2}$
0.420	$(0.318 \pm 0.011) \times 10^{-2}$	$(0.322 \pm 0.012) \times 10^{-2}$
0.435	$(0.379 \pm 0.012) \times 10^{-2}$	$(0.381 \pm 0.013) \times 10^{-2}$
0.450	$(0.579 \pm 0.035) \times 10^{-2}$	$(0.581 \pm 0.036) \times 10^{-2}$
0.465	$(0.677 \pm 0.038) \times 10^{-2}$	$(0.677 \pm 0.039) \times 10^{-2}$
0.480	$(0.719 \pm 0.041) \times 10^{-2}$	$(0.709 \pm 0.043) \times 10^{-2}$
0.495	$(0.901 \pm 0.044) \times 10^{-2}$	$(0.888 \pm 0.047) \times 10^{-2}$
0.510	$(0.101 \pm 0.005) \times 10^{-1}$	$(0.099 \pm 0.005) \times 10^{-1}$
0.525	$(0.117 \pm 0.005) \times 10^{-1}$	$(0.114 \pm 0.005) \times 10^{-1}$
0.540	$(0.137 \pm 0.005) \times 10^{-1}$	$(0.134 \pm 0.006) \times 10^{-1}$
0.555	$(0.156 \pm 0.005) \times 10^{-1}$	$(0.150 \pm 0.007) \times 10^{-1}$
0.570	$(0.155 \pm 0.011) \times 10^{-1}$	$(0.145 \pm 0.012) \times 10^{-1}$
0.585	$(0.183 \pm 0.012) \times 10^{-1}$	$(0.169 \pm 0.014) \times 10^{-1}$
0.600	$(0.194 \pm 0.012) \times 10^{-1}$	$(0.178 \pm 0.014) \times 10^{-1}$
0.615	$(0.201 \pm 0.012) \times 10^{-1}$	$(0.178 \pm 0.015) \times 10^{-1}$
0.630	$(0.246 \pm 0.013) \times 10^{-1}$	$(0.215 \pm 0.017) \times 10^{-1}$
0.645	$(0.230 \pm 0.011) \times 10^{-1}$	$(0.195 \pm 0.018) \times 10^{-1}$
0.660	$(0.248 \pm 0.011) \times 10^{-1}$	$(0.203 \pm 0.020) \times 10^{-1}$
0.675	$(0.249 \pm 0.011) \times 10^{-1}$	$(0.190 \pm 0.022) \times 10^{-1}$
0.690	$(0.310 \pm 0.022) \times 10^{-1}$	$(0.241 \pm 0.032) \times 10^{-1}$
0.705	$(0.349 \pm 0.022) \times 10^{-1}$	$(0.267 \pm 0.035) \times 10^{-1}$
0.720	$(0.316 \pm 0.021) \times 10^{-1}$	$(0.223 \pm 0.038) \times 10^{-1}$
0.735	$(0.297 \pm 0.021) \times 10^{-1}$	$(0.178 \pm 0.041) \times 10^{-1}$
0.750	$(0.312 \pm 0.016) \times 10^{-1}$	
0.765	$(0.361 \pm 0.016) \times 10^{-1}$	
0.780	$(0.381 \pm 0.017) \times 10^{-1}$	
0.795	$(0.382 \pm 0.023) \times 10^{-1}$	
0.810	$(0.369 \pm 0.023) \times 10^{-1}$	
0.825	$(0.429 \pm 0.024) \times 10^{-1}$	
0.840	$(0.461 \pm 0.024) \times 10^{-1}$	
0.855	$(0.439 \pm 0.024) \times 10^{-1}$	
0.870	$(0.429 \pm 0.024) \times 10^{-1}$	
0.885	$(0.415 \pm 0.025) \times 10^{-1}$	

Coulomb distortion is not negligible even at these energies. In the few-hundred MeV region used in the past for determinations of response functions, the Coulomb effect must be rather large.

IV. EXAMPLES

To illustrate the extrapolation procedure, we show in Figs. 3–5 the response functions for a few extreme cases of (q, ω) as a function of $A^{-1/3}$. The point for $A=4$,

TABLE V. Nuclear matter response as a function of energy loss. The second column gives the cross sections defined in Eq. (5); the third column gives the cross sections, corrected for nucleon excitations, defined by Eq. (6). $E = 3.595$ GeV. $\theta = 25^\circ$.

ω (GeV)	$d\sigma/d\Omega d\omega$ ($\mu\text{b/sr GeV}$)	$d\sigma/d\Omega d\omega_{\text{sub}}$ ($\mu\text{b/sr GeV}$)
0.405	$(0.137 \pm 0.040) \times 10^{-4}$	$(0.124 \pm 0.036) \times 10^{-4}$
0.420	$(0.218 \pm 0.043) \times 10^{-4}$	$(0.219 \pm 0.043) \times 10^{-4}$
0.435	$(0.205 \pm 0.046) \times 10^{-4}$	$(0.210 \pm 0.047) \times 10^{-4}$
0.450	$(0.284 \pm 0.052) \times 10^{-4}$	$(0.292 \pm 0.053) \times 10^{-4}$
0.465	$(0.290 \pm 0.054) \times 10^{-4}$	$(0.294 \pm 0.056) \times 10^{-4}$
0.480	$(0.476 \pm 0.061) \times 10^{-4}$	$(0.481 \pm 0.062) \times 10^{-4}$
0.495	$(0.415 \pm 0.067) \times 10^{-4}$	$(0.412 \pm 0.068) \times 10^{-4}$
0.510	$(0.743 \pm 0.080) \times 10^{-4}$	$(0.739 \pm 0.083) \times 10^{-4}$
0.525	$(0.910 \pm 0.091) \times 10^{-4}$	$(0.893 \pm 0.093) \times 10^{-4}$
0.540	$(0.123 \pm 0.024) \times 10^{-3}$	$(0.121 \pm 0.024) \times 10^{-3}$
0.555	$(0.178 \pm 0.026) \times 10^{-3}$	$(0.174 \pm 0.027) \times 10^{-3}$
0.570	$(0.142 \pm 0.026) \times 10^{-3}$	$(0.132 \pm 0.026) \times 10^{-3}$
0.585	$(0.212 \pm 0.030) \times 10^{-3}$	$(0.200 \pm 0.031) \times 10^{-3}$
0.600	$(0.232 \pm 0.032) \times 10^{-3}$	$(0.215 \pm 0.033) \times 10^{-3}$
0.615	$(0.261 \pm 0.030) \times 10^{-3}$	$(0.235 \pm 0.032) \times 10^{-3}$
0.630	$(0.322 \pm 0.033) \times 10^{-3}$	$(0.289 \pm 0.035) \times 10^{-3}$
0.645	$(0.426 \pm 0.058) \times 10^{-3}$	$(0.382 \pm 0.061) \times 10^{-3}$
0.660	$(0.427 \pm 0.071) \times 10^{-3}$	$(0.368 \pm 0.074) \times 10^{-3}$
0.675	$(0.414 \pm 0.073) \times 10^{-3}$	$(0.339 \pm 0.078) \times 10^{-3}$
0.690	$(0.677 \pm 0.084) \times 10^{-3}$	$(0.584 \pm 0.090) \times 10^{-3}$
0.705	$(0.905 \pm 0.096) \times 10^{-3}$	$(0.798 \pm 0.105) \times 10^{-3}$
0.720	$(0.115 \pm 0.009) \times 10^{-2}$	$(0.102 \pm 0.010) \times 10^{-2}$
0.735	$(0.111 \pm 0.009) \times 10^{-2}$	$(0.095 \pm 0.011) \times 10^{-2}$
0.750	$(0.137 \pm 0.010) \times 10^{-2}$	$(0.118 \pm 0.012) \times 10^{-2}$
0.765	$(0.192 \pm 0.018) \times 10^{-2}$	$(0.168 \pm 0.020) \times 10^{-2}$
0.780	$(0.204 \pm 0.019) \times 10^{-2}$	$(0.177 \pm 0.021) \times 10^{-2}$
0.795	$(0.201 \pm 0.019) \times 10^{-2}$	$(0.167 \pm 0.023) \times 10^{-2}$
0.810	$(0.220 \pm 0.021) \times 10^{-2}$	$(0.176 \pm 0.026) \times 10^{-2}$
0.825	$(0.296 \pm 0.023) \times 10^{-2}$	$(0.245 \pm 0.029) \times 10^{-2}$
0.840	$(0.276 \pm 0.018) \times 10^{-2}$	$(0.212 \pm 0.027) \times 10^{-2}$
0.855	$(0.332 \pm 0.020) \times 10^{-2}$	$(0.253 \pm 0.030) \times 10^{-2}$
0.870	$(0.388 \pm 0.031) \times 10^{-2}$	$(0.299 \pm 0.041) \times 10^{-2}$
0.885	$(0.449 \pm 0.032) \times 10^{-2}$	$(0.346 \pm 0.045) \times 10^{-2}$
0.900	$(0.418 \pm 0.033) \times 10^{-2}$	$(0.292 \pm 0.049) \times 10^{-2}$
0.915	$(0.436 \pm 0.034) \times 10^{-2}$	$(0.285 \pm 0.055) \times 10^{-2}$
0.930	$(0.482 \pm 0.034) \times 10^{-2}$	$(0.311 \pm 0.060) \times 10^{-2}$
0.945	$(0.497 \pm 0.026) \times 10^{-2}$	$(0.298 \pm 0.062) \times 10^{-2}$
0.960	$(0.463 \pm 0.026) \times 10^{-2}$	
0.975	$(0.548 \pm 0.038) \times 10^{-2}$	
0.990	$(0.607 \pm 0.040) \times 10^{-2}$	
1.005	$(0.543 \pm 0.040) \times 10^{-2}$	
1.020	$(0.639 \pm 0.038) \times 10^{-2}$	
1.035	$(0.595 \pm 0.040) \times 10^{-2}$	
1.050	$(0.708 \pm 0.032) \times 10^{-2}$	
1.065	$(0.726 \pm 0.032) \times 10^{-2}$	
1.080	$(0.818 \pm 0.051) \times 10^{-2}$	
1.095	$(0.799 \pm 0.051) \times 10^{-2}$	
1.110	$(0.815 \pm 0.053) \times 10^{-2}$	
1.125	$(0.850 \pm 0.054) \times 10^{-2}$	
1.140	$(0.919 \pm 0.054) \times 10^{-2}$	
1.155	$(0.997 \pm 0.056) \times 10^{-2}$	

TABLE VI. Nuclear matter response as a function of energy loss. The second column gives the cross sections defined in Eq. (5); the third column gives the cross sections, corrected for nucleon excitations, defined by Eq. (6). $E = 3.595$ GeV, $\theta = 30^\circ$.

ω (GeV)	$d\sigma/d\Omega d\omega$ ($\mu\text{b/sr GeV}$)	$d\sigma/d\Omega d\omega_{\text{sub}}$ ($\mu\text{b/sr GeV}$)
0.645	$(0.349 \pm 0.083) \times 10^{-5}$	$(0.310 \pm 0.077) \times 10^{-5}$
0.660	$(0.546 \pm 0.095) \times 10^{-5}$	$(0.503 \pm 0.091) \times 10^{-5}$
0.675	$(0.561 \pm 0.089) \times 10^{-5}$	$(0.547 \pm 0.091) \times 10^{-5}$
0.690	$(0.746 \pm 0.109) \times 10^{-5}$	$(0.723 \pm 0.111) \times 10^{-5}$
0.705	$(0.719 \pm 0.146) \times 10^{-5}$	$(0.675 \pm 0.149) \times 10^{-5}$
0.720	$(0.901 \pm 0.165) \times 10^{-5}$	$(0.837 \pm 0.169) \times 10^{-5}$
0.735	$(0.139 \pm 0.022) \times 10^{-4}$	$(0.130 \pm 0.023) \times 10^{-4}$
0.750	$(0.134 \pm 0.017) \times 10^{-4}$	$(0.120 \pm 0.017) \times 10^{-4}$
0.765	$(0.178 \pm 0.020) \times 10^{-4}$	$(0.161 \pm 0.020) \times 10^{-4}$
0.780	$(0.209 \pm 0.023) \times 10^{-4}$	$(0.185 \pm 0.023) \times 10^{-4}$
0.795	$(0.302 \pm 0.039) \times 10^{-4}$	$(0.272 \pm 0.039) \times 10^{-4}$
0.810	$(0.303 \pm 0.037) \times 10^{-4}$	$(0.262 \pm 0.038) \times 10^{-4}$
0.825	$(0.368 \pm 0.041) \times 10^{-4}$	$(0.315 \pm 0.042) \times 10^{-4}$
0.840	$(0.428 \pm 0.045) \times 10^{-4}$	$(0.360 \pm 0.047) \times 10^{-4}$
0.855	$(0.483 \pm 0.049) \times 10^{-4}$	$(0.396 \pm 0.051) \times 10^{-4}$
0.870	$(0.703 \pm 0.059) \times 10^{-4}$	$(0.600 \pm 0.062) \times 10^{-4}$
0.885	$(0.781 \pm 0.063) \times 10^{-4}$	$(0.653 \pm 0.067) \times 10^{-4}$
0.900	$(0.768 \pm 0.113) \times 10^{-4}$	$(0.606 \pm 0.119) \times 10^{-4}$
0.915	$(0.143 \pm 0.017) \times 10^{-3}$	$(0.126 \pm 0.017) \times 10^{-3}$
0.930	$(0.146 \pm 0.016) \times 10^{-3}$	$(0.123 \pm 0.017) \times 10^{-3}$
0.945	$(0.161 \pm 0.017) \times 10^{-3}$	$(0.134 \pm 0.018) \times 10^{-3}$
0.960	$(0.218 \pm 0.020) \times 10^{-3}$	$(0.186 \pm 0.022) \times 10^{-3}$
0.975	$(0.201 \pm 0.044) \times 10^{-3}$	$(0.185 \pm 0.020) \times 10^{-3}$
0.990	$(0.304 \pm 0.044) \times 10^{-3}$	$(0.246 \pm 0.049) \times 10^{-3}$
1.005	$(0.316 \pm 0.050) \times 10^{-3}$	$(0.237 \pm 0.056) \times 10^{-3}$
1.020	$(0.422 \pm 0.060) \times 10^{-3}$	$(0.328 \pm 0.067) \times 10^{-3}$
1.035	$(0.398 \pm 0.064) \times 10^{-3}$	$(0.279 \pm 0.072) \times 10^{-3}$
1.050	$(0.370 \pm 0.061) \times 10^{-3}$	$(0.222 \pm 0.073) \times 10^{-3}$
1.065	$(0.527 \pm 0.072) \times 10^{-3}$	$(0.348 \pm 0.086) \times 10^{-3}$
1.080	$(0.642 \pm 0.050) \times 10^{-3}$	$(0.442 \pm 0.076) \times 10^{-3}$
1.095	$(0.770 \pm 0.053) \times 10^{-3}$	$(0.554 \pm 0.085) \times 10^{-3}$
1.110	$(0.813 \pm 0.069) \times 10^{-3}$	$(0.531 \pm 0.103) \times 10^{-3}$
1.125	$(0.798 \pm 0.071) \times 10^{-3}$	$(0.502 \pm 0.115) \times 10^{-3}$
1.140	$(0.105 \pm 0.008) \times 10^{-2}$	$(0.067 \pm 0.013) \times 10^{-2}$
1.155	$(0.100 \pm 0.008) \times 10^{-2}$	$(0.058 \pm 0.014) \times 10^{-2}$
1.170	$(0.105 \pm 0.006) \times 10^{-2}$	
1.185	$(0.115 \pm 0.007) \times 10^{-2}$	
1.200	$(0.122 \pm 0.010) \times 10^{-2}$	
1.215	$(0.136 \pm 0.010) \times 10^{-2}$	
1.230	$(0.112 \pm 0.010) \times 10^{-2}$	
1.245	$(0.136 \pm 0.011) \times 10^{-2}$	
1.260	$(0.161 \pm 0.010) \times 10^{-2}$	
1.275	$(0.193 \pm 0.010) \times 10^{-2}$	
1.290	$(0.165 \pm 0.020) \times 10^{-2}$	
1.305	$(0.192 \pm 0.021) \times 10^{-2}$	
1.320	$(0.195 \pm 0.020) \times 10^{-2}$	
1.335	$(0.220 \pm 0.021) \times 10^{-2}$	
1.350	$(0.218 \pm 0.015) \times 10^{-2}$	
1.365	$(0.261 \pm 0.016) \times 10^{-2}$	
1.380	$(0.259 \pm 0.021) \times 10^{-2}$	
1.395	$(0.270 \pm 0.022) \times 10^{-2}$	
1.410	$(0.279 \pm 0.022) \times 10^{-2}$	
1.425	$(0.372 \pm 0.024) \times 10^{-2}$	
1.440	$(0.296 \pm 0.018) \times 10^{-2}$	
1.455	$(0.314 \pm 0.018) \times 10^{-2}$	
1.470	$(0.341 \pm 0.028) \times 10^{-2}$	
1.485	$(0.316 \pm 0.029) \times 10^{-2}$	

TABLE VI. (Continued).

ω (GeV)	$d\sigma/d\Omega d\omega$ ($\mu\text{b/sr GeV}$)	$d\sigma/d\Omega d\omega_{\text{sub}}$ ($\mu\text{b/sr GeV}$)
1.500	$(0.350 \pm 0.029) \times 10^{-2}$	
1.515	$(0.385 \pm 0.021) \times 10^{-2}$	
1.530	$(0.433 \pm 0.022) \times 10^{-2}$	
1.545	$(0.431 \pm 0.031) \times 10^{-2}$	
1.560	$(0.450 \pm 0.031) \times 10^{-2}$	
1.575	$(0.471 \pm 0.031) \times 10^{-2}$	
1.590	$(0.529 \pm 0.033) \times 10^{-2}$	
1.605	$(0.553 \pm 0.025) \times 10^{-2}$	
1.620	$(0.595 \pm 0.036) \times 10^{-2}$	
1.635	$(0.637 \pm 0.038) \times 10^{-2}$	
1.650	$(0.583 \pm 0.037) \times 10^{-2}$	
1.665	$(0.650 \pm 0.037) \times 10^{-2}$	
1.680	$(0.664 \pm 0.031) \times 10^{-2}$	
1.695	$(0.639 \pm 0.051) \times 10^{-2}$	
1.710	$(0.671 \pm 0.049) \times 10^{-2}$	
1.725	$(0.749 \pm 0.049) \times 10^{-2}$	
1.740	$(0.752 \pm 0.050) \times 10^{-2}$	
1.755	$(0.793 \pm 0.039) \times 10^{-2}$	
1.770	$(0.763 \pm 0.038) \times 10^{-2}$	
1.785	$(0.855 \pm 0.054) \times 10^{-2}$	
1.800	$(0.791 \pm 0.054) \times 10^{-2}$	
1.815	$(0.861 \pm 0.054) \times 10^{-2}$	
1.830	$(0.104 \pm 0.004) \times 10^{-1}$	
1.845	$(0.994 \pm 0.065) \times 10^{-2}$	
1.860	$(0.101 \pm 0.006) \times 10^{-1}$	
1.875	$(0.853 \pm 0.061) \times 10^{-2}$	
1.890	$(0.103 \pm 0.007) \times 10^{-1}$	
1.905	$(0.104 \pm 0.005) \times 10^{-1}$	
1.920	$(0.114 \pm 0.007) \times 10^{-1}$	
1.935	$(0.111 \pm 0.007) \times 10^{-1}$	
1.950	$(0.108 \pm 0.006) \times 10^{-1}$	
1.965	$(0.111 \pm 0.005) \times 10^{-1}$	
1.980	$(0.103 \pm 0.006) \times 10^{-1}$	
1.995	$(0.112 \pm 0.007) \times 10^{-1}$	
2.010	$(0.113 \pm 0.007) \times 10^{-1}$	
2.025	$(0.116 \pm 0.007) \times 10^{-1}$	

which was not used for the extrapolation, nearly always deviates from the straight line defined by $A = 12-197$. The remaining points lie on a straight line, within the statistical and systematical errors of the data.

The slope of the fit changes as a function of q and ω . This results from several effects:

1. The quasielastic peak gets wider as the Fermi momentum k_F increases with A . This tends to increase $\Sigma(q, \omega)$ in the wings of the quasielastic peak; $\Sigma(q, \omega)$ decreases near the peak.

2. The quasielastic peak shifts towards larger energy loss with increasing A due to the increasing average nucleon separation energy. This tends to make the slope more positive for $\omega < q^2/2m$.

3. Spreading of the Δ peak becomes more important at larger k_F , A . This leads to an increase in $\Sigma(q, \omega)$ for large A and $\omega > q^2/2m$.

The response functions of gold and nuclear matter

differ typically by 10%. In extreme cases, differences up to 25% are found.

V. RESULTS

In Tables I–VII we list the extrapolated inclusive cross sections per nucleon for nuclear matter with $N = Z$. The first column is based on the data as measured; the second one gives values corrected for inelastic processes on individual nucleons.

The error bars given reflect both the experimental errors and the ones introduced due to the extrapolation. An additional systematic error of 5% should be added to account for the normalization error of the original data.

Figs. 6 and 7 give a qualitative impression of the data and their evolution as a function of q, ω . As pointed out earlier, the extrapolated inclusive response function for

TABLE VII. Nuclear matter response as a function of energy loss. The second column gives the cross sections defined in Eq. (5); the third column gives the cross sections, corrected for nucleon excitations, defined by Eq. (6). $E = 3.995$ GeV, $\theta = 30^\circ$.

ω (GeV)	$d\sigma/d\Omega d\omega$ ($\mu\text{b}/\text{sr GeV}$)	$d\sigma/d\Omega d\omega_{\text{sub}}$ ($\mu\text{b}/\text{sr GeV}$)
0.810	$(0.704 \pm 0.348) \times 10^{-6}$	$(0.634 \pm 0.355) \times 10^{-6}$
0.825	$(0.147 \pm 0.033) \times 10^{-5}$	$(0.139 \pm 0.033) \times 10^{-5}$
0.840	$(0.159 \pm 0.035) \times 10^{-5}$	$(0.146 \pm 0.035) \times 10^{-5}$
0.855	$(0.238 \pm 0.045) \times 10^{-5}$	$(0.220 \pm 0.046) \times 10^{-5}$
0.870	$(0.224 \pm 0.042) \times 10^{-5}$	$(0.196 \pm 0.042) \times 10^{-5}$
0.885	$(0.237 \pm 0.043) \times 10^{-5}$	$(0.197 \pm 0.043) \times 10^{-5}$
0.900	$(0.245 \pm 0.040) \times 10^{-5}$	$(0.191 \pm 0.041) \times 10^{-5}$
0.915	$(0.491 \pm 0.067) \times 10^{-5}$	$(0.425 \pm 0.068) \times 10^{-5}$
0.930	$(0.579 \pm 0.076) \times 10^{-5}$	$(0.491 \pm 0.078) \times 10^{-5}$
0.945	$(0.711 \pm 0.108) \times 10^{-5}$	$(0.599 \pm 0.111) \times 10^{-5}$
0.960	$(0.824 \pm 0.098) \times 10^{-5}$	$(0.676 \pm 0.099) \times 10^{-5}$
0.975	$(0.873 \pm 0.101) \times 10^{-5}$	$(0.684 \pm 0.104) \times 10^{-5}$
0.990	$(0.128 \pm 0.017) \times 10^{-4}$	$(0.105 \pm 0.017) \times 10^{-4}$
1.005	$(0.169 \pm 0.021) \times 10^{-4}$	$(0.141 \pm 0.021) \times 10^{-4}$
1.020	$(0.144 \pm 0.018) \times 10^{-4}$	$(0.107 \pm 0.019) \times 10^{-4}$
1.035	$(0.174 \pm 0.020) \times 10^{-4}$	$(0.129 \pm 0.021) \times 10^{-4}$
1.050	$(0.235 \pm 0.024) \times 10^{-4}$	$(0.181 \pm 0.025) \times 10^{-4}$
1.065	$(0.279 \pm 0.023) \times 10^{-4}$	$(0.214 \pm 0.025) \times 10^{-4}$
1.080	$(0.319 \pm 0.025) \times 10^{-4}$	$(0.240 \pm 0.027) \times 10^{-4}$
1.095	$(0.391 \pm 0.027) \times 10^{-4}$	$(0.297 \pm 0.031) \times 10^{-4}$
1.110	$(0.479 \pm 0.038) \times 10^{-4}$	$(0.368 \pm 0.043) \times 10^{-4}$
1.125	$(0.571 \pm 0.042) \times 10^{-4}$	$(0.440 \pm 0.048) \times 10^{-4}$
1.140	$(0.634 \pm 0.044) \times 10^{-4}$	$(0.478 \pm 0.052) \times 10^{-4}$
1.155	$(0.858 \pm 0.053) \times 10^{-4}$	$(0.679 \pm 0.063) \times 10^{-4}$
1.170	$(0.954 \pm 0.051) \times 10^{-4}$	$(0.741 \pm 0.066) \times 10^{-4}$
1.185	$(0.112 \pm 0.006) \times 10^{-3}$	$(0.087 \pm 0.007) \times 10^{-3}$
1.200	$(0.134 \pm 0.006) \times 10^{-3}$	$(0.104 \pm 0.009) \times 10^{-3}$
1.215	$(0.177 \pm 0.011) \times 10^{-3}$	$(0.143 \pm 0.013) \times 10^{-3}$
1.230	$(0.199 \pm 0.011) \times 10^{-3}$	$(0.159 \pm 0.014) \times 10^{-3}$
1.245	$(0.202 \pm 0.011) \times 10^{-3}$	$(0.154 \pm 0.015) \times 10^{-3}$
1.260	$(0.239 \pm 0.012) \times 10^{-3}$	$(0.184 \pm 0.017) \times 10^{-3}$
1.275	$(0.289 \pm 0.013) \times 10^{-3}$	$(0.225 \pm 0.020) \times 10^{-3}$
1.290	$(0.311 \pm 0.011) \times 10^{-3}$	$(0.238 \pm 0.020) \times 10^{-3}$
1.305	$(0.342 \pm 0.011) \times 10^{-3}$	$(0.255 \pm 0.022) \times 10^{-3}$
1.320	$(0.406 \pm 0.019) \times 10^{-3}$	$(0.305 \pm 0.030) \times 10^{-3}$
1.335	$(0.431 \pm 0.019) \times 10^{-3}$	$(0.315 \pm 0.033) \times 10^{-3}$
1.350	$(0.491 \pm 0.021) \times 10^{-3}$	$(0.357 \pm 0.037) \times 10^{-3}$
1.365	$(0.501 \pm 0.022) \times 10^{-3}$	
1.380	$(0.547 \pm 0.024) \times 10^{-3}$	
1.395	$(0.596 \pm 0.023) \times 10^{-3}$	
1.410	$(0.635 \pm 0.027) \times 10^{-3}$	
1.425	$(0.669 \pm 0.039) \times 10^{-3}$	
1.440	$(0.716 \pm 0.042) \times 10^{-3}$	

nuclear matter for $N \neq Z$ has also been determined; it is available from the first author,

VI. LOW-ENERGY RESPONSE

For a systematic study of the nuclear response as a function of q and ω , lower- q data are desirable as well. An experiment,¹⁴ performed 15 years ago at Stanford, provides a set of data for many nuclei with $A = 6 \div 208$. This large body of data, taken at 500 MeV and 60° , also allows an extrapolation to nuclear matter. The response

for nuclear matter from this set of data is given in Table VIII. These data, of poorer quality than the very recent high- q data, exhibit fluctuations in the slopes [Eq. (7)], originating in the statistical errors of the data. The consequences of such fluctuations for the extrapolation can be largely suppressed in the following way. As the average slope changes as a function of ω , as previously discussed, we determined for every bin ω the relative slope [Eq. (7)] averaged over the region $\delta\omega = \pm 20$ MeV around this bin, and used these sliding values for the extrapolation of the center bin.

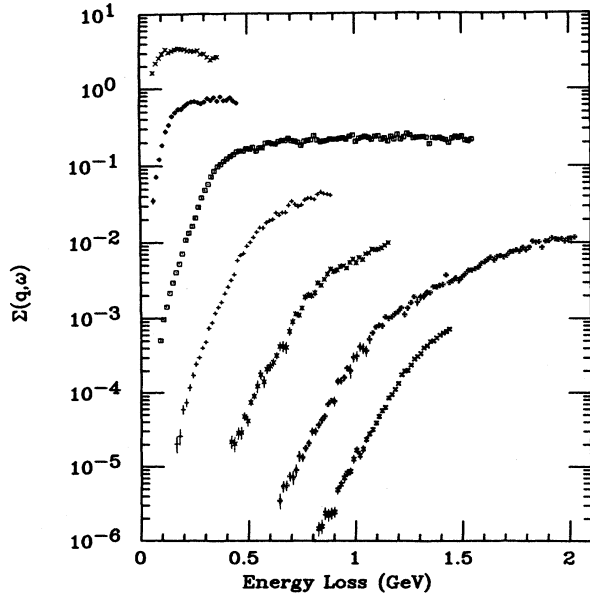


FIG. 6. Cross section per nucleon for nuclear matter as a function of q and ω . The sets of data correspond to, respectively, incident energy/scattering angle of 2.0 GeV/15°, 2.0/20°, 3.6/16°, 3.6/20°, 3.6/25°, 3.6/30°, and 4.0/30°.

For an incident energy as low as 500 MeV, and the corresponding lower final energies, Coulomb distortion is more important. We show in Fig. 8 the response function calculated with and without Coulomb corrections.

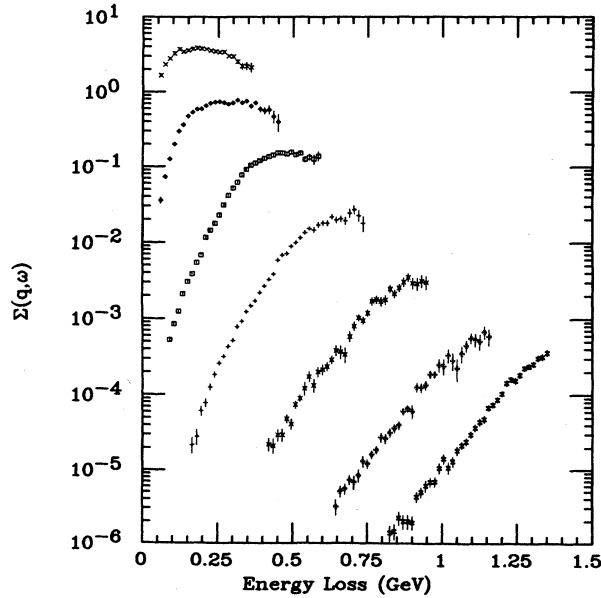


FIG. 7. Same as Fig. 6, but with inelastic excitation of the nucleon removed.

TABLE VIII. Nuclear matter response for the 500 MeV, 500° data, defined according to Eq. (5). $E = 0.500$ GeV, $\theta = 60^\circ$.

ω (GeV)	$d\sigma/d\Omega d\omega$ ($\mu\text{b/sr GeV}$)
0.303	(0.457 \pm 0.028)
0.293	(0.420 \pm 0.025)
0.283	(0.398 \pm 0.024)
0.273	(0.362 \pm 0.022)
0.263	(0.341 \pm 0.020)
0.253	(0.333 \pm 0.019)
0.244	(0.329 \pm 0.018)
0.234	(0.325 \pm 0.016)
0.224	(0.342 \pm 0.015)
0.214	(0.359 \pm 0.015)
0.204	(0.376 \pm 0.015)
0.194	(0.415 \pm 0.016)
0.184	(0.441 \pm 0.018)
0.174	(0.460 \pm 0.020)
0.165	(0.477 \pm 0.022)
0.155	(0.475 \pm 0.023)
0.145	(0.467 \pm 0.027)
0.135	(0.456 \pm 0.025)
0.125	(0.431 \pm 0.024)
0.114	(0.426 \pm 0.024)
0.105	(0.406 \pm 0.022)
0.095	(0.394 \pm 0.020)
0.086	(0.359 \pm 0.018)
0.076	(0.316 \pm 0.015)
0.066	(0.257 \pm 0.013)
0.056	(0.216 \pm 0.015)
0.046	(0.163 \pm 0.013)
0.036	(0.117 \pm 0.013)

VII. NUCLEAR MATTER DENSITY

The preceding response functions correspond to nuclear matter as defined by the extrapolation of finite nuclei. For a comparison to theory, we also need to determine the corresponding density of nuclear matter, ρ_{NM} .

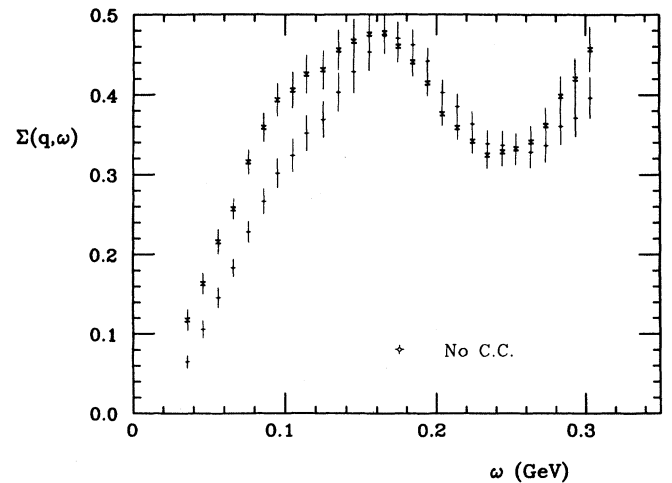


FIG. 8. Cross section for nuclear matter, with and without Coulomb corrections, for 500 MeV and 60°.

The charge density $\bar{\rho}_c$ of nuclei in the interior, constant-density region of nuclei, depends weakly on A . The nucleon density $\bar{\rho}_m$ can be obtained from $\bar{\rho}_c$ under the assumption that proton and neutron densities have nearly the same radius. The differences between neutron and proton radii are estimated from Hartree-Fock calculations, and the scatter is included in the error bar. These matter densities can be extrapolated to $A = \infty$ to remove the weak A dependence, yielding a nuclear matter density of $0.162 \pm 0.005 \text{ fm}^{-3}$. This value is lower than the often quoted value of 0.17 fm^{-3} , but agrees with values obtained from fits of nuclear charge densities using the liquid drop model, or Hartree-Fock calculations with effective NN forces adjusted to experimental charge densities.^{15,16}

Alternatively, one can determine the nuclear matter density from a measurement of the nuclear Fermi momentum, which is related in nuclear matter to ρ by $\bar{\rho} = 2k_F^3/3\pi^2$. This Fermi momentum can be determined from dynamical properties of nuclei, i.e., nucleon momentum distributions as measured by inclusive electron scattering. These values of k_F for finite nuclei must again be extrapolated to $A = \infty$ to remove the effects of the nuclear surface. The data of Whitney *et al.*¹⁴ allow a determination of k_F for a series of nuclei $A = 6-208$. At the comparatively low momentum transfer (2.5 fm^{-1}) of these data, final-state interaction effects of the recoil nucleon are important. Brieva and Dellafiore¹⁷ have fitted these data within the framework of the Fermi gas model using an energy-dependent optical potential to describe final-state interactions (FSI). They find values of k_F typically 10% lower than when neglecting FSI. Extrapolat-

ing these values to $A = \infty$ yields $264 \pm 10 \text{ MeV}/c$. The value of $0.161 \pm 0.016 \text{ fm}^{-3}$ for the nucleon density corresponding to this k_F agrees with the foregoing value of $0.162 \pm 0.005 \text{ fm}^{-3}$ determined from charge distributions.

VIII. CONCLUSION

In this paper, we have determined the response function of nuclear matter, by extrapolating to $A = \infty$ the inclusive cross sections $\sigma(q, \omega)$ measured for a set of nuclei $A = 4-197$. We find that this extrapolation can be performed in a reliable way. The differences between a heavy nucleus and nuclear matter are appreciable, as a consequence of the fact that even for a heavy nucleus more than 50% of the nucleons are in the surface region of the nucleus where the density is lower than the one in the central region, i.e., nuclear matter. These differences typically amount to 10%; in the most interesting region, at comparatively low energy loss, larger differences up to 30% are found.

The set of data for nuclear matter provided in this paper is limited to the particular kinematic region where a coherent set of data for many nuclei has been measured. These data do, however, cover a kinematic region which is of particular interest for the study of nuclear matter properties. The region of large momentum transfer and comparatively small energy transfer studied is sensitive to properties of nuclear matter at large momentum (up to several times k_F). At the same time, the energy transfers and recoil-nucleon momenta are large enough to ease the description of the final state, thus allowing for a quantitative study of nuclear matter properties.

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